EVANESCENT WAVE METHOD FOR PROPAGATION IN GRADED INDEX SLAB WAVEGUIDES

E. Navon and L.B. Felsen
Polytechnic Institute of New York
Route 110, Farmingdale, N.Y. 11735

Abstract

The evanescent wave method employed previously for construction of high-frequency asymptotic modal solutions in symmetric slab waveguides with unbounded analytic refractive index profiles is generalized to account for an inhomogeneous guiding region embedded in a homogeneous exterior medium. Analytical and numerical results are presented for various classes of profiles.

Summary

High-frequency propagation in graded index waveguides is generally analyzed by ray or WKB methods, which are based on the tracking of local plane waves with real phase. These methods have the disadvantage of failing near the caustics (turning points) of the modal ray system and of being inaccurate for lower order modes. A recently developed new method, the evanescent wave method, furnishes a high-frequency asymptotic solution based on the tracking of local inhomogeneous (evanescent) plane waves with complex phase. By this procedure, one avoids consideration of caustics since these then generally lie in a complex coordinate space. The evanescent wave method has been applied to the study of guided modes in various transversely unbounded symmetric refractive index profiles in two[1] and three[2] dimensions. The solutions so generated, which are required to decay at infinity in the transverse (x) domain, have been shown to agree term by term with the asymptotic expansion of exact solutions, when available for special cases, thereby providing confidence in the validity of the method.

The objective of the present investigation is to extend the evanescent wave method to the treatment of polynomial type symmetric slab index profiles with a bounded inhomogeneous interior (core) region $|x| < x_c$ and a homogeneous exterior region. The solution in the core region must now be constructed in terms of two linearly independent functions, which can be chosen as decaying and growing, respectively, with $|x|$. In the homogeneous exterior, the solution is given as a decaying exponential. The decaying function in the core is a generalization of the decaying solution for the unbounded profile ($x_c \to \infty$), which alone is relevant in that case to assure proper behavior of the transverse modal field at $x \to \infty$. In the unbounded profile, the transverse modal field $\varphi_\ell(x)$ and the corresponding axial propagation coefficient β_ℓ depend on a parameter ℓ, which is restricted to integer (or zero) values and has therefore been identified as the mode index[1]. No such restriction applies to the generalized forms for the bounded profile. The quired generalization can be achieved on replacing ℓ in the asymptotic solutions for $\varphi_\ell(x) \equiv \varphi(x, \ell)$ and $\beta_\ell \equiv \beta(\ell)$ by a non-integral parameter $\nu = \ell + \Delta\nu_\ell$. These asymptotic solutions then have the form:

$$\varphi_\nu(x) \sim \exp\left[-kI(x)\right] \sum_{m=0}^{\infty} A_{m\nu}(x) k^{-m} \tag{1a}$$

$$\beta_\nu^2 \sim k^2 \sum_{m=0}^{\infty} B_{m\nu}\, k^{-m} \tag{1b}$$

where k denotes the free-space wavenumber. The decay function $I(x) > 0$, the x-dependent expansion coefficients $A_{m\nu}(x)$, and the constant coefficients $B_{m\nu}$ are evaluated systematically by the evanescent wave method for unbounded profiles[1]. Defining the refractive index n(x) in the core as $n^2(x) = n_0^2 - a_0^2 \overline{n}^2(x)$, where n_0 and a_0 are constants and $\overline{n}^2(x)$ is normalized so that $\overline{n}(x) \sim x$ as $x \to 0$, we then construct the asymptotic form of the linearly independent growing solution $\psi_\nu(x)$ from a knowledge of $\varphi_\nu(x)$ by subjecting ν and the profile parameter a_0 to a certain transformation that keeps β_ν invariant for both solutions. The latter constraint is required if $\varphi_\nu(x)$ and $\psi_\nu(x)$ are to be employed for synthesis of a single modal field. The two linearly independent asymptotic solutions must be normalized so that their linear superposition ensures even or odd x-symmetry of the modal field, as required in a symmetric index profile. Finally, the remaining requirement of continuity of the field and its x-derivative across the core boundary x_c leads to a dispersion equation that determines the modified mode parameter $\nu = \ell + \Delta\nu_\ell$.

Following the above scheme, one may employ the evanescent wave method to construct the asymptotic expansion of the modal field solution in a graded index waveguide with a symmetrically bounded core solely from a knowledge of the asymptotic expansion of the (unnormalized) decaying solution in the unbounded profile. To achieve this result, use has been made of exact solutions for two bounded special profiles, the parabolic and hyperbolic tangent. While the former has been studied previously[3], although by a less convenient procedure than employed by us, our solution for the latter is new. Both profiles are utilized to establish the transformation that yields the growing solution from the decaying solution, and to select the normalization coefficients in their linear combination. The selection is based on a comparison of the asymptotic fields for a general polynomial profile with the asymptotic expansion of the exact solution for the parabolic profile near the x=0 plane since all profiles considered here behave parabolically near that plane. The implications of this "locally parabolic" comparison method are discussed in connection with the exact asymptotic solution for the hyperbolic tangent profile.

Numerical results for the perturbation $\Delta\nu_\ell$ of the mode parameter ℓ due to the presence of the core boundary are presented for parabolic and polynomial profiles and are compared with data obtained in the literature by other techniques.

References

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